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The Removal of Methyl Violet from Water by Solvent Sublation

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ABSTRACT

Methyl violet (MV. $C_{14}H_{28}N_3Cl$), a cationic dye, was removed from an aqueous solution by solvent sublation of methyl violet-sodium dodecylbenzulfonate (DBS) complex (sublate) into 2-pentanol. A stoichiometric amount of surfactant (surfactant:dye = 1:1) was the most effective for the removal, with over 97% MV removed from the aqueous solution in 10 min by solvent sublation. The removal rate was somewhat enhanced by higher airflow rates and almost independent of the volume of the organic solvent floated on the top of the aqueous column. The effects of electrolytes (e.g., KCl), non-hydrophobic organics (e.g., ethanol), and pH of the solution on the process were studied. The comparison of air stripping and solvent extraction with the solvent sublation process was made. The solvent sublation process followed first-order kinetics.

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A characteristic parameter, apparent activation energy of attachment of the sublate to bubbles, was estimated at a value of 8.19 kJ/mol.

INTRODUCTION

Solvent sublation, originated by Sebba[1] for ionic-surfactant complexes, has shown a promise for removing hydrophobic molecular compounds and ion-pair complexes from aqueous systems. Solvent sublation is one among the several adsorptive bubble separation techniques in which a hydrophobic compound is levitated on a bubble surface to the top of an aqueous column where they encounter a solvent layer (e.g., mineral oil, lauryl alcohol) to which the material is transferred as the bubbles move through the solvent layer.[2,3]

Solvent sublation process is different from solvent extraction. When fine gas bubbles (of very small radii in the range of 0.01–0.05 cm and in laminar flow) pass through an aqueous column (with an overlaying organic layer) containing hydrophobic compound, because of their inherent tendency to concentrate at the air–water interface, these hydrophobic materials would be collected on the bubble surface by diffusion through the thin boundary layer surrounding the air bubble. As the bubble transits the aqueous column and moves through the organic solvent layer, the adsorbed phase gets stripped into the organic phase. Thus, the solvent sublation process improves the efficiency of a bubble aeration column. Moreover, the presence of the organic solvent also reduces the eventual redispersion of the material into the aqueous column upon bubble bursting, which usually occurs in conventional bubble aeration columns.[4]

The solvent sublation techniques have been studied by some workers. Most of these studies have been focused on the removal of organic pollutants from aqueous systems, such as alkyl phthalate, volatile chlorinate organics, dichlorobenzenes, nitrophenols, polynuclear aromatics and chlorinated pesticides,[5–11] and the emission of volatile organic compounds (volatile chlorinated organics and Toluene) to atmosphere in the solvent sublation.[12,13] Other studies are the solvent sublation of dyes in the aqueous solution, such as removal of bromophenol blue from water by solvent sublation with hexadecyl-pyridiumchloride into iso-pentanol,[14] the separation of methyl orange from Rhodamine B,[15,16] the solvent sublation of methyl orange–hexadecyltrimethyl ammonium ion pair,[17] magenta (a cationic dye) with sodium lauryl sulfate[18] and Direct Red and Acid Red (two anionic dyes) with hexadecyltrimethylammonium ion.[19,20]

Recently, both mechanism and kinetics of solvent sublation were investigated by Palagyi et al.[21–25] They have proposed an ionic associate formation mechanism and second-order kinetics in the separation of iodide from water by solvent sublation with CPC (N-cetylpyridinium chloride) into benzene. However, the studies of both kinetics and thermodynamics of the solvent sublation are few. The removal of MV, a cationic dye, from an aqueous solution by solvent sublation was studied. Because MV can cause some environmental problem, the search of a simple and effective removal method is necessary. In this article, the kinetics and thermodynamics of the solvent sublation of MV-DBS were investigated and the efficiencies of solvent sublation with some conventional separation methods were compared. The effects of parameters, such as the airflow rates and the volume ratio of aqueous to the organic solvent, were studied, and the effects of certain co-solutes were also tested. In the solvent sublation system, 2-pentanol was chosen as the organic solvent, which has a high solubility of the MV-DBS, immiscible to the aqueous phase, low-aqueous–organic solvent interfacial tension, non-toxic, and non-volatile.

EXPERIMENTAL

The solvent sublation system was similar to that described earlier.[26] The glass column was 90 cm in length and 3.7 cm in inside diameter with three access ports. A titanium plate (pore size 5–10 μm) was used to introduce air bubbles into the aqueous phase. Airflow rate was measured with a soap flowmeter. To obtain a representative concentration of the aqueous phase, the sample solution was withdrawn from the middle access for analysis.

Reagent-grade sodium dodecylbenzensulfonate (Shangai Chemical Agents Factory, China) was used as a collector without further purification. Reagent-grade methyl violet, 2-pentanol (Shanghai Chemical Agents Factory, China), and the other agents were all analytical grade.

For the solvent sublation runs, first, DBS was added to the sample solution (400 mL) to form the dye-surfactant complex, followed by co-solute, such as ethanol and KCl. Second, the solution containing 1.58×10^{-5} mol/L MV was poured into the sublation column, and 10 mL 2-pentanol was added immediately; then the timer was started and the samples of aqueous solution were taken for analysis at a certain time.

The pH of the solution was measured with a pHs-3C (Shaihai Rex Industry, China). UV–visible spectra of the sample solutions were measured with a Unico PC2100 UV/Vis Spectrophotometer (Unico Com., China) at the maximum peak of 582 nm.

RESULTS AND DISCUSSION

Effect of the Molar Ratio of DBS to MV on Solvent Sublation

The effect of DBS concentration on the solvent sublation of MV is shown in Fig. 1. It was found that a 1:1 mole ratio of surfactant to dye gave the fastest rate of separation and the lowest residual dye concentration, with over 97% of MV being removed in 10 min. At a smaller concentration of surfactant, the rate of removal was slower and the level of the residual dye was greater, presumably due to incomplete formation of a dye-surfactant complex. But when the ratio was higher than 1:1, the rate of solvent sublation was smaller and the removal efficiency was lower, somewhat presumably due to the competition of the bubble surface by the excess surfactant ion with the dye-surfactant complex. To find the effect of the surfactant itself on the solvent sublation of MV, an experiment was carried out. For the process of solvent sublation of surfactant DBS, the recovery efficiency was higher than 98%,[27] which indicated that the surfactant was competing for the adsorbed spots of bubbles. The excess surfactant also caused the emulsification of the iso-pentanol (which was observed during the solvent sublation process with a larger excess of surfactant), such that the dye-surfactant complex in the iso-pentanol was constantly dispersed back into the solution and the separation efficiency decreased largely. The optimal separation efficiency with surfactant

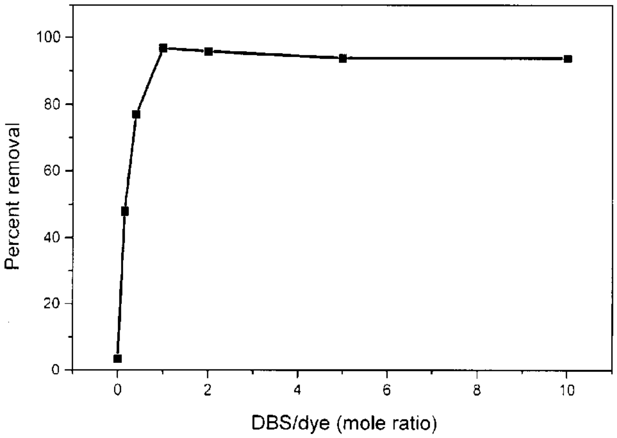

Figure 1. Effect of DBS/dye ratio dose on solvent sublation. $C = 1.58 \times 10^{-5}$ mol/L, $V_w = 400$ mL, $V_o = 10$ mL, rate of airflow = 65 mL/min, and duration of airflow = 10 min.

dosage at the stoichiometric amount was also observed for the solvent sublation of HTA-Acid Red.[20] It was noted that this finding was quite different from the results of the solvent sublation of methylene blue and methyl orange studied by Wilson et al. and Karger et al.,[15–17,28] who found that the rate of removal of dyes increased with the increase of the surfactant concentration when it was much in excess of the stoichiometric amount. This contradiction was probably due to the difference in the formation constant of the varied complexes of surfactant-dye and may be also due to the altered molar ratio of the surfactant and dye in the complexes.[20]

Effect of Different Airflow Rates on Solvent Sublation

The solvent sublation of MV into iso-pentanol was investigated at three different airflow rates (30, 60, and 105 mL/min). It was observed that the removal rates increased with the increase of airflow rates, as shown in Fig. 2. However, the increase in removal rate was out of proportion to the increase of airflow rates, similar to the results by Valsaraj et al.[4] This was probably explained that with increasing airflow rates the mean bubble radius increased,

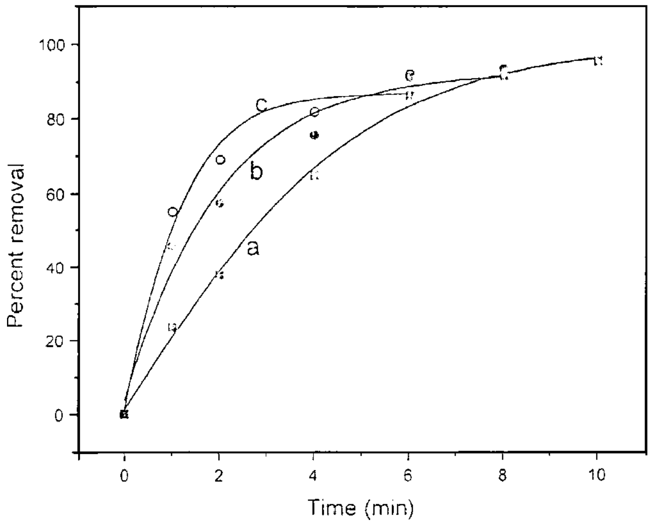

Figure 2. Effect of rate of airflow on solvent sublation. $C = 1.58 \times 10^{-5}$ mol/L, ratio of DBS to MV = 1:1, $V_w = 400$ mL, $V_o = 10$ mL, a: 30; b: 60; and c: 105 mL/min.

thus the interfacial area per unit volume of air (which is given by 3/r) decreased, and the bubble residence time also reduced because larger bubbles had higher rise velocities. Furthermore, the axial dispersion certainly increased with the increase of the airflow rates, which would impair the performance of the sublation process.

It was observed that at higher flow rates the oil–water interface was drastically disrupted, and some drops of the top layer could return to the solution. Although the increased airflow rate can improve the removal rate of solvent sublation, the removal efficiency would decrease if the airflow rate was quite high, for the air currents arising from the quite high airflow rates would disrupt the oil–aqueous interface.

Effect of the Volume Ratio of Organic Solvent to Aqueous for Solvent Sublation

Sebba[1] has shown that in the solvent sublation of ion-surfactant complexes from aqueous solution into 2-octanol, the removal efficiency is independent of the amount of the volume of 2-octanol. Caballero has made a conclusion that the sublation efficiency is independent on the organic solvent volume only until the saturation of the phase occurs by the sublate.[29] We achieved the same result with these experiments, which are shown in Fig. 3. It is observed that the removal efficiency is about 90% without the organic solvent, which showed the organic solvent was not a decisive factor. No significant improvement in removal efficiency was observed when the volume of 2-pentanol exceeded 2.5 mL in the 400-mL solution. The results showed that sublation was independent of the volume of organic solvent until the saturation of the phase occurs by the sublate. Generally, mass transfer occurs from gas bubbles crossing the aqueous–solvent interface and not from diffusion of solute across this interface; the amount of material transferred should depend only on the amount of air crossing the interface and not on the organic volume. In liquid–liquid extraction, the volume ratio of the two immiscible phases is a very important parameter, which is an important difference between the liquid–liquid extraction and solvent sublation. But if the organic volume used in solvent sublation is too low, the oil–water interface will be drastically disrupted at a high airflow rate, and the process will lose its efficiency. Hence, the airflow rates and solvent volume must be chosen to keep the minimal disruption of the interface.

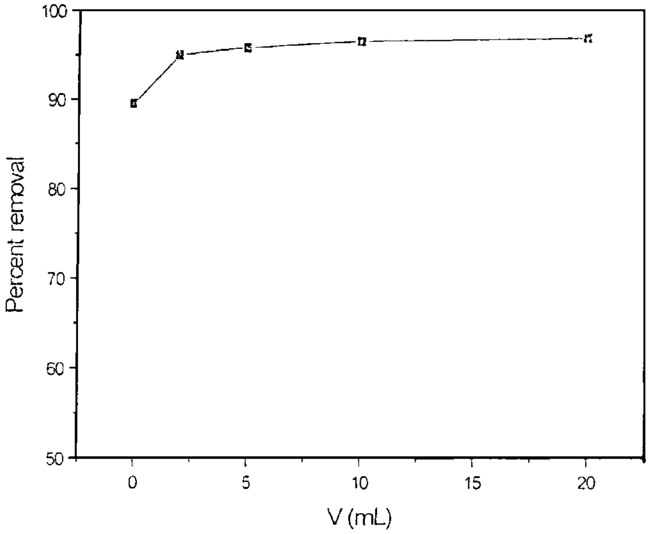

Figure 3. Effect of solvent volume on solvent sublation. $C = 1.58 \times 10^{-5}$ mol/L, ratio of DBS to MV = 1:1, $V_w = 400$ mL, and rate of airflow = 65 mL/min.

Effects of Co-solute

The influence of various mole fractions of ethanol used as co-solute ranging from 0.005 to 0.04 upon the removal rates of MV is shown in Fig. 4. The low mole fraction of ethanol enhanced removal rates, whereas high mole fraction of ethanol decreased the removal rates.

The enhancement in removal rates might be due to two factors. First we noticed that addition of ethanol changed the bubble properties considerably; the number of very small bubbles was much lager than when ethanol was absent. This is a well-known effect arising from the lowering of surface tension of water, which prevents the bubbles from growing to large sizes. So these smaller bubbles provide a very large surface area per unit volume of air, which contributes to enhance mass transfer from the liquid phase to the bubbles. At the same time, smaller bubbles also have slower rise velocities.[30] However, higher mole fraction of ethanol decrease the removal rate due to the increase of the solubility of sublate.

The effect of inorganic salt KCl used as co-solute on the sublation of MV-DBS complex is shown in Fig. 5. Increasing the KCl concentration tended to decrease the removal rate and the removal efficiency. This was attributed to an

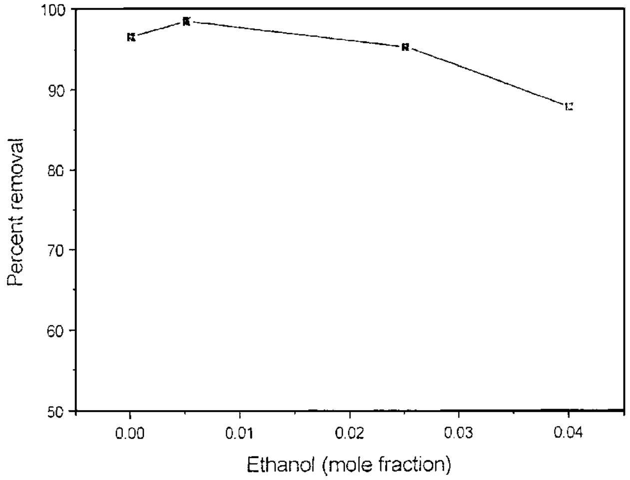

Figure 4. Effect of mole fraction of ethanol on solvent sublation. $C = 1.58 \times 10^{-5}\,\mathrm{mol/L}$, ratio of DBS to MV = 1:1, $V_w = 400\,\mathrm{mL}$, $V_o = 10\,\mathrm{mL}$, and rate of airflow = 65 mL/min.

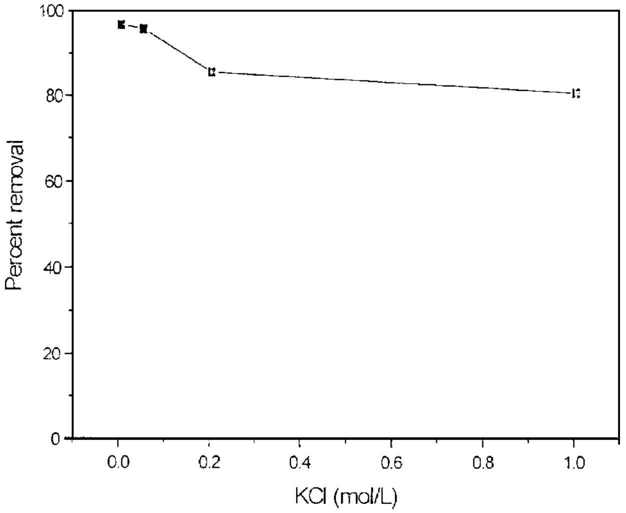

Figure 5. Effect of KCl concentration on solvent sublation. $C = 1.58 \times 10^{-5}\,\mathrm{mol/L}$, ratio of DBS to MV = 1:1, $V_w = 400\,\mathrm{mL}$, $V_o = 10\,\mathrm{mL}$, rate of airflow = 65 mL/min, and duration of airflow = 10 min.

ion-pair equilibrium that existed in the aqueous solution between the dye and surfactant molecules. In aqueous solution, there existed the equilibrium:

$$HP^{+}B^{-} + K^{+}Cl^{-} \rightleftharpoons HP^{+}Cl^{-} + K^{+}B^{-}$$

where HP^{-} represents $C_{14}H_{28}N_3^{-}$ (dye cationic ion), and B^{-} represents dodecylbenzensulfonate anionic ion. It can be seen that from the equation that the increase of the salt concentration (e.g., KCl) drove the equilibrium toward a large concentration of $HP^{-}Cl^{-}$, which was hydrophilic, and less mole DBS-MV complex (i.e., $HP^{-}B^{-}$) existed in the aqueous phase. As a result, the rate of removal decreased.

Effect of pH

The effect of pH on the removal of DBS-MV in the process of solvent sublation is shown in Fig. 6. The removal rate and removal efficiency of MV increased at higher pH. Over 90% of MV was removed from the solution by solvent sublation in 10 min at the natural pH value. But at lower pH value, the removal rate and efficiency decreased.

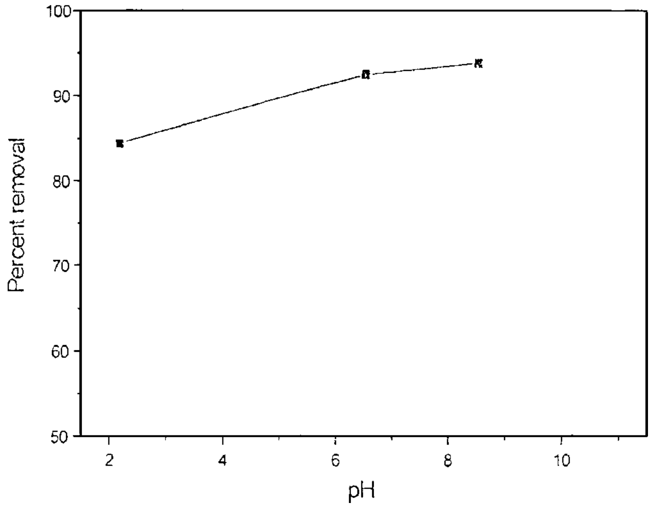

Figure 6. Effect of pH on solvent sublation. $C = 1.58 \times 10^{-5}$ mol/L, ratio of DBS to MV = 1:1, $V_w = 400$ mL, $V_o = 10$ mL, rate of airflow = 65 mL/min, and duration of airflow = 10 min.

Comparison with Air Stripping and Solvent Extraction

The air-stripping process of MV (without DBS) was carried out. It was observed that the removal efficiency of the air stripping was very low (4%), but that of the solvent sublation increased largely (97%) at the same experiment conditions. From the above experiments, a conclusion can be drawn that the surface activity could accelerate the removal rate largely, i.e., the surface activity is a decisive factor in the solvent sublation process of anionic surfactant–cationic dye.

To obtain more accurate results for comparative purposes, a separate extraction process was carried out by using 250-mL extraction funnels. The experiment of the solvent extraction of 50 mL MV in the absence of DBS into 10-mL 2-pentanol was carried out. The result showed that the MV was almost not removed (5%), which was similar to that of the air-stripping process (4%). MV-DBS solution (50 mL) and 10 mL 2-pentanol were, respectively, poured into the funnel keeping stirring the aqueous layer; the equilibrium was reestablished in about 3 days. It was found that the removal efficiency of the solvent extraction process was similar to that of the solvent sublation process. But the former process takes a long time; the latter only spent 10 min to reach equilibrium. It can be observed that in the solvent extraction process there existed immiscible drops, which made the equilibrium slow. At the same time, the rate of solvent extraction was dependent on the volume ratio of organic and aqueous phase. From the above comparisons it is shown that the process of the solvent sublation is more rapid and more convenient than that of the solvent extraction; the addition of surfactant is very important to the removal of MV.

Kinetics and Thermodynamics of the Solvent Sublation

In the chemical reaction, the rate is followed by:

$$-\mathrm{d}c/\mathrm{d}t = \mathrm{k}c^{n} \tag{1}$$

where c is the reaction concentration, k is the apparent rate constant, and n is the orders of chemical reaction. Upon the analysis of the experiment results, we found that the kinetics of solvent sublation process was obedient to the equation:

$$-\mathrm{d}c/\mathrm{d}t = \mathrm{k}c \tag{2}$$

It showed that the kinetics mechanism of solvent sublation process was consistent with the first-order kinetics equation.

In the solvent sublation process, first the MV-DBS molecules and the bubbles attracted, and then the repulsive forces increased, which would result in the boundary layer compressed and become thinner and then eclipse finally. The change of energy of this process was similar to that of the chemical reaction process.[4] So we could use the Arrhenius equation to describe the process:

$$\mathrm{lnk} = E\sqrt{(\mathrm{RT})} + \mathrm{B} \tag{3}$$

where k is the apparent rate constant, R is the common gas constant, B is the integrate constant, and E is the solvent sublation apparent activation energy. The relationship of $\ln \mathrm{k}$ and $1/T$ is linear, and the value of the apparent active energy can be calculated from the slope of the line. With the same concentration of MV-DBS solution, the experiment temperature was set at 290K, 318K, and 345K. The kinetics of MV at different temperature is shown in Fig. 7. It was observed that the rate of solvent sublation was decreased with the increase of temperature. The relationship of $\ln \mathrm{k}$ vs. $1/T$ is linear, which was shown in Fig. 8. The value of apparent active energy was calculated as 8.19 kJ/mol by the slope of Fig. 8, which was so small that the solvent sublation process was very rapid.

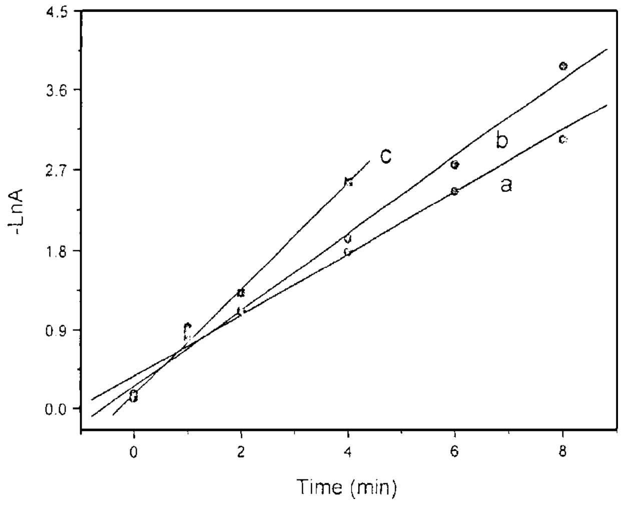

Figure 7. Kinetics of solvent sublation with different temperature. C = 1.58×10^{-5} mol/L, ratio of DBS to MV = 1:1, $V_w = 400$ mL, $V_o = 10$ mL, rate of airflow = 65 mL/min, a: 345K; b: 318K; c: 290K.

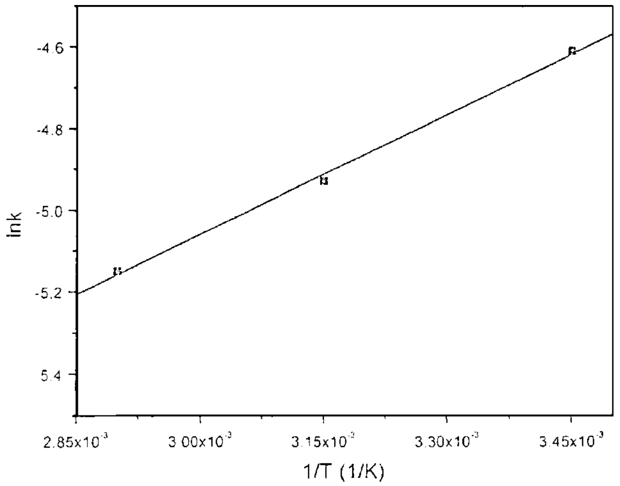

Figure 8. Curve of ln k vs. $1/T$; the data are from Fig. 7.

CONCLUSIONS

MV was removed effectively from the aqueous solution by solvent sublation with an anionic surfactant, DBS. A stoichiometric amount of surfactant (dye:surfactant = 1:1) was found to be the most effective where over 97% of MV was removed in 10 min. The increased airflow rates enhance the process of solvent sublation, provided that the bubbles size is kept small. However, at higher airflow rates, the generation of axial dispersion would compromise the efficiency of the solvent sublation.

The solvent sublation was somewhat independent of the organic volume, provided that the organic volume is larger than a critical value, i.e., disruption of organic–aqueous interface was minimal.

Smaller fraction of ethanol (0.005) enhanced the efficiency of the solvent sublation; by contrast, larger fraction of ethanol (0.04) decreased the efficiency of removal. The increase of the KCl concentration would decrease the removal rate of sublation for it made the ion equation move toward the hydrophilic product direction. The natural pH value is the optimal to the higher removal efficiency, but the lower pH value decreased the removal efficiency.

The kinetics of the solvent sublation was followed by first-order kinetics. The apparent active energy was put forward as a parameter of the solvent sublation and was calculated as 8.19 kJ/mol.

REFERENCES

1. Sebba, F. *Ion Flotation*; Elsevier: New York, 1962; 112–123.
2. Lemlich, R. *Adsorptive Bubble Separation Techniques*; Academic: New York, 1972.
3. Carleson, T.F. Adsorptive bubble separation process. In *Surfactant-Based Separation Process*; Scamehorn, J.T., Harwell, J.H., Eds.; Dekker: New York, 1989.
4. Valsaraj, K.T.; Portor, J.L.; Liljenfeldt, E.K.; Springer, C. Solvent sublation for the removal of hydrophobic chlorinated compounds from aqueous solutions. Water Res. **1986**, *20* (9), 1161–1175.
5. Clarke, A.N.; Wilson, D.J. Separation by flotation. Sep. Purif. Methods **1978**, *7* (1), 55–98.
6. Wilson, D.J.; Pearson, D.E. *Solvent Sublation of Organic Contaminants for Water Reclamation*, Report RU-83 6; Bureau of Reclamation. U.S., Department of Interior: Washington, DC, 1984.
7. Tamamushi, K.; Wilson, D.J. Removal of refractory organics by aeration. VI solvent sublation of alkyl phthalates. Sep. Sci. Technol. **1984**, *19* (13), 1013–1023.
8. Lionel, T.; Wilson, D.J.; Pearson, D.E. Removal of refractory organics by aeration. I methyl chloroform. Sep. Sci. Technol. **1981**, *16* (8), 907–935.
9. Wilson, D.J.; Valsaraj, K.T. Removal of refractory organics by aeration. III a fast algorithm for modeling solvent sublation columns. Sep. Sci. Technol. **1983**, *17* (12), 1387–1396.
10. Huang, S.D.; Valsaraj, K.T.; Wilson, D.J. Removal of refractory organics by aeration. V solvent sublation of naphthalene and phenanthrene. Sep. Sci. Technol. **1983**, *18* (10), 941–968.
11. Valsaraj, K.T.; Wilson, D.J. Removal of refractory organics by aeration. IV solvent sublation of chlorinated organics and nitrophenols. J. Colloid Surf. **1983**, *8* (2), 203–224.
12. Ososkov, V.; Kebbekus, B.; Chou, C.C. Emission of volatile organic compounds to the atmosphere in the solvent sublation process. I toluene. Sep. Sci. Technol. **1996**, *31* (2), 213–227.
13. Ososkov, V.; Kebbekus, B.; Chen, M. Emission of volatile organic compounds to the atmosphere in the solvent sublation process. II volatile chlorinated organic compounds. Sep. Sci. Technol. **1996**, *31* (10), 1377–1391.
14. Lu, Y.J.; Zhu, X.H. Removal of bromophenol blue from water by solvent sublation. Sep. Sci. Technol. **2001**, *36* (16), 3763–3776.

15. Carger, A.B.; Karger, B.L. Use of rate phenomena in solvent sublation separation of methyl orange and rhodamine B. Anal. Chem. **1966**, *38* (4), 652–654.
16. Karger, B.L.; Carger, A.B.; Lee, S.B. Sep. Sci. **1967**, *2*, 39.
17. Karger, B.L.; Pinfold, T.A. Studies in the mechanism of sublate removal by solvent sublation. Part I. Sep. Sci. **1970**, *5* (5), 603–617.
18. Sheu, G.L.; Huang, S.D. Solvent sublation and adsorbing colloid flotation of magenta. Sep. Sci. Technol. **1987**, *22* (11), 2253–2262.
19. Huang, J.Y.; Huang, S.D. Solvent sublation and adsorbing colloid flotation of acid red. Sep. Sci. Technol. **1991**, *26* (1), 59–71.
20. Cheng, M.H.; Huang, S.D. Solvent sublation and adsorbing colloid flotation of direct red. J. Colloid Interface Sci. **1988**, *126* (1), 346–354.
21. Palagyi, S. Kinetics of transport extraction based on solvent sublation. Czech. J. Phys. **1999**, *49* (2), 739–745.
22. Palagyi, S. Separation and preconcentration of ionic solutes by transport extraction based on solvent sublation. Chem. Pap.-Chem. Zvesti **1998**, *52* (5), 671–681.
23. Palagyi, S.; Braun, T. Transport extraction for trace-element separation and preconcentration. 3. Polyurethane foam sorbent supported transport extraction. Fresenius' J. Anal. Chem. **1994**, *348* (3), 211–214.
24. Palagyi, S.; Braun, T. Transport extraction for trace-element separation and preconcentration. 2. Preconcentration of iodine from aqueous-solutions. Fresenius' J. Anal. Chem. **1994**, *346* (10–11), 905–908.
25. Palagyi, S.; Braun, T. Transport extraction for trace-element separation and preconcentration. 1. Comparison with solvent-extraction. Fresenius' J. Anal. Chem. **1993**, *345* (6), 415–419.
26. Ren, X.; Chen, W.G.; Zhu, X.H. A study on separation mechanism of solvent sublation. Acta Sci. Nat. Univ. Sunyatseni **1996**, *35* (6), 24–28.
27. Lu, Y.J.; Wang, Y.S.; Xiong, Y.; Zhu, X.H. Study on the kinetics and thermodynamics of surfactants in solvent sublation. Fresenius' J. Anal. Chem. **2001**, *370* (8), 1071–1076.
28. Womack, J.L.; Lichter, L.C.; Wilson, D.J. Removal of refractory organics by aeration. II solvent sublation of methylene blue and methyl orange. Sep. Sci. Technol. **1982**, *17* (7), 897–924.
29. Caballero, M.; Cela, R.; Perez-Bustamante, J.A. Analytical applications of some flotation techniques—a review. Talanta **1990**, *37* (2), 275–300.
30. Levich, V.G. *Physicochemical Hydrodynamics*; Prentice Hall: Englewood Cliff, NJ, 1962; 434.

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